10.3.1 ELECTRICITY

10.3.1.1 Electric Forces and Fields

10.3.1.1.1 Electric Charge

The American polymath Benjamin Franklin (1706–1790) named the two different kinds of charge *positive* and *negative*. Positive and negative charges are said to be opposite because an object with an equal amount of positive and negative charge has no net charge. Furthermore:

- like charges repel each other;
- unlike charges attract each other.

Electric charge can be neither created nor destroyed, an observation that leads to one of the fundamental laws of nature, The Principle of Conservation of Electric Charge.

In 1909, the American physicist Robert Millikan (1886–1953) demonstrated that electric charge was *quantized*, existing as multiples of a fundamental unit of charge, e, now known to be the charge of a single electron or proton. The value of e has since been determined to be 1.602×10^{-19} C, where the *coulomb* (C) is the SI unit of electric charge.

Materials in which electric charges move freely, such as copper and aluminium, are called **conductors**. Most metals are conductors. Materials in which electric charges do not move freely, such as glass, rubber and plastic, are called **insulators**.

Semiconductors are a third class of materials characterised by electrical properties that lie between those of insulators and conductors. Certain metals also belong to a fourth class of materials called *superconductors*. Superconductors become perfect conductors when they are cooled below a certain temperature.

Both conductors and insulators can be charged by contact, by rubbing a copper rod with a wool cloth, or a glass rod with a silk cloth, for example (see Section 7.3.1 Electrostatics for a more detailed treatment of the *triboelectric effect*). Conductors can also be charged by **induction**.

A surface charge can also be induced on an insulator by polarization.

10.3.1.1.2 Electric Force

In the 1780s, French physicist Charles Augustin de Coulomb (1736–1806) conducted a series experiments that led to the development of Coulomb's Law, which states that the electric force ($F_{electric}$) between two charges is proportional to the magnitude of the charges (q_1 & q_2) and inversely proportional to the square of the distance (r) between them:

$$F_{electric} = k_C \frac{q_1 q_2}{r^2}$$

where k_C is the Coulomb constant (8.9875×10^9 N·m²/C²). Note that $F_{electric}$ is a vector quantity and must be treated accordingly. The resultant electric force on any charge is the vector sum of the individual electric forces on that charge.

Note also that electric force is a field force, like the gravitational force between two bodies, and as such the mathematical representation of Coulomb's Law is very similar to that of Newton's Universal Law of Gravitation:

$$F_g = G\frac{m_1 m_2}{r^2}$$

10.3.1.1.3 The Electric Field

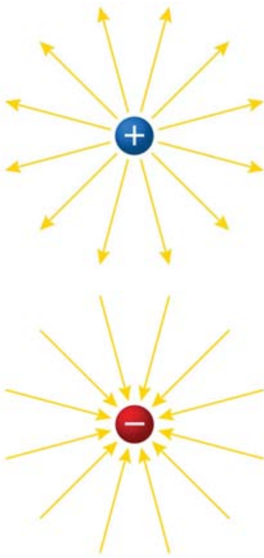
An **electric field** exists in the region around any charged object. Any other charged object entering this field will interact with it.

While the strength of an electric field, E, can be defined as the magnitude of the electric force ($F_{electric}$) acting on a test charge, q_0, divided by the charge of q_0, it is more generally defined in terms of the magnitude of the charge (q) producing the field and the distance (r) between that charge and the point at which the field strength is being measured:

$$E = \frac{F_{electric}}{q_0} = \frac{k_C q}{r^2}$$

Because electric field strength is a ratio of force to charge, the SI units of E are newtons per coulomb (N/C).

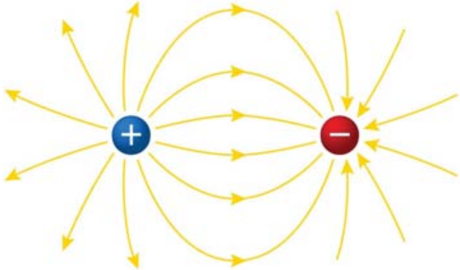
By convention, the direction of the electric field vector, **E**, is the direction in which an electric force would act on a positive test charge. If q is positive, the field due to this charge is directed outward, radially from q. If q is negative, the field is directed towards q. As with electric force, the electric field due to more than one charge is calculated by applying the *principle of superposition*.



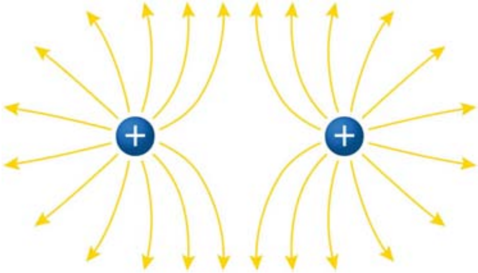
Field lines around positive and negative charges

A convenient aid for visualising electric field patterns is to draw **electric field lines** pointing in the direction of the electric field. By convention, electric field lines are drawn according to the following rules:

- The lines must begin on positive charges or at infinity and must terminate on negative charges or at infinity;
- The electric field vector, **E**, is tangent to the lines at any point;
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge;
- No two field lines from the same field can cross each other.



Field lines around unlike charges



Field lines around like charges

A good electric conductor, such as copper, contains charges (electrons) that are not bound to any atoms and are free to move about within the material. When no net motion of charge is occurring within a conductor, the conductor is said to be in *electrostatic equilibrium*. Such a conductor has the following four properties:

- The electric field is zero everywhere inside the conductor;
- Any excess charge on an isolated conductor resides entirely on the conductor's outer surface;
- The electric field just outside a charged conductor is perpendicular to the conductor's surface;
- On an irregularly shaped conductor, charge tends to accumulate where the radius of curvature of the surface is smallest, that is, its sharp points.

10.3.1.2 Electrical Energy and Capacitance

10.3.1.2.1 Electrical Potential Energy

When two charges interact, there is an electric force between them. As with the gravitational force associated with an object's position relative to Earth, there is a potential energy associated with this force. This kind of potential energy is called **electrical potential energy**. Unlike gravitational potential energy, electrical potential energy results from the interaction of two objects' charges, not their masses.

Electric potential energy is a form of mechanical energy.

Electrical potential energy can be associated with a charge in a uniform field (i.e. one that has the same value and direction at all points). As with other forms of potential energy, it is the difference in electrical potential energy between a point in the field and some reference point, or zero level. Thus, the electrical potential energy of a charge in a uniform electric field is given by the equation:

$$PE_{electric} = -qEd$$

where q is the charge, E is the electric field strength and d is the displacement from the reference point in the direction of the field.

This equation, however, is valid only for a uniform electric field. When dealing with the interaction between two or more point charges, we need to consider the fact that the electric field involved in not uniform. Thus, the electrical potential energy associated with a pair of charges is given by the alternate equation:

$$PE_{electric} = k_C \frac{q_1 q_2}{r}$$

where k_C is the Coulomb constant, q_1 and q_2 are the charges, and r is the distance between them.

The SI unit for electrical potential energy is the *joule* (J).

10.3.1.2.2 Potential Difference

While electrical potential energy varies with the magnitude of the charge involved, we will find it more practical in the study of electricity to have some measure that is independent of the charge involved. To this end we define the **electric potential** at a point as follows:

$$V = \frac{PE_{electric}}{q}$$

Because the reference point for measuring electrical potential energy is arbitrary, the reference point for measuring electric potential energy is also arbitrary. Thus, only changes in electric potential are significant. The **potential difference** between two points can then be defined as follows:

$$\Delta V = \frac{\Delta PE_{electric}}{q}$$

In a uniform electric field, potential difference varies with the displacement from the reference point:

$$\Delta V = -E\Delta d$$

The reference point for potential difference near a point charge, however, is usually at infinity, and the potential difference between a point at infinity and a point near a point charge is given by the following equation:

$$\Delta V = k_C \frac{q}{r}$$

The superposition principle can be used to calculate the electric potential for a group of charges.

The SI derived unit for electrical potential difference is the *volt* (V), after the Italian physicist Count Alessandro Volta (1745–1827), who invented the voltaic pile, possibly the first chemical battery. A potential difference of one volt is equivalent to a difference of one joule of energy per coulomb of charge (1 V = 1 J/C). Potential difference is often referred to simply as *voltage*, and algebraically simply as V (rather than ΔV).

10.3.1.2.3 Capacitance

A capacitor is a device that is used to store electrical potential energy, or charge. A typical design for a capacitor consists of two parallel metal plates, separated by a dielectric material (*i.e.* an insulator) so that charge cannot be transferred directly between the two plates. A capacitor can then be *charged* by moving electric charge from one plate to the other, via an appropriate external circuit. Once charged, a capacitor can discharge if its plates are connected by a conducting path.

The ability of a given capacitor to store charge is known as its **capacitance**, defined as the ratio of the net charge on each plate (Q) to the potential difference (V) created by the separated charges as follows:

$$C = \frac{Q}{V}$$

Capacitance depends on the shape of the capacitor, the distance between the plates, and the dielectric between the plates.

The SI unit for charge is the *farad* (F), which is equivalent to one coulomb per volt (C/V). In practice, most typical capacitors have capacitances ranging from microfarads (1μF = 1 × 10^{-6} F) to picofarads (1pF = 1 × 10^{-12} F).

A charged capacitor stores electrical potential energy because it requires work to move charges through a circuit to the opposite plates of a capacitor, and the potential energy stored in a charged capacitor depends on the charge and the final potential difference between the capacitor's two plates:

$$PE_{electric} = \tfrac{1}{2}QV$$

10.3.1.3 Current and Resistance

10.3.1.3.1 Electric Current

An electric current exists whenever there is a net movement of charge through a medium. The current is the rate at which these charges move through the cross section of the wire. If ΔQ is the amount of charge that passes through this area in a time interval (Δt), then the current (I) is the ratio of the amount of charge to the time interval:

$$I = \frac{\Delta Q}{\Delta t}$$

The SI unit for current is the *ampere* (A). One ampere is equivalent to one coulomb of charge passing through a cross-sectional area in a time interval of one second (1 A = 1 C/s).

The moving charges that make up a current can be positive, negative, or a combination of the two. In a common conductor, such as copper, current is due to the motion of negatively charged electrons. In certain particle accelerators a current exists when positively charged protons are set in motion. In some cases—in gases and dissolved salts, for example—current is the result of positive charges moving in one direction and negative charges moving in the opposite direction. Positive and negative charges in motion are sometimes called *charge carriers*.

Conventional current is defined as the current consisting of positive charge that would have the same effect as the actual motion of the charge carriers, regardless of whether the charge carriers are positive, negative, or a combination of the two.

Drift velocity is the net velocity of charge carriers. It is a common misconception that electrons travel very rapidly in a conductor. In fact, when a potential difference is applied across a conductor, an electric field is established almost instantaneously, setting charges in motion and creating a current, but the charges themselves travel much more slowly. Their path is essentially random, with a gradual movement against the direction of the electric field (*e.g.* in a copper wire carrying a current of 10 A, the drift velocity is 2.46×10^{-4} m/s).

There are also two different types of current: **direct current** (dc) and **alternating current** (ac). In direct current, charges move in only one direction. In alternating current, the motion of charges continuously changes in the forward and reverse directions—they simply vibrate back and forth. Batteries are a typical source of direct current, and while generators can produce either direct or alternating current, those used by power utilities typically produce the latter because its characteristics make it better suited to power transmission.

10.3.1.3.2 Resistance

In 1827, as part of his theory of electricity, German physicist Georg Ohm (1789–1854) reported that he had noted *a direct proportionality between the potential difference applied across a conductor and the resultant electric current*, an observation now known as **Ohm's Law** and stated as follows:

$$\frac{V}{I} = \text{constant}$$

Indeed, although most materials can be classified as conductors or insulators, some conductors allow charges to move through them more easily than others. The opposition to the motion of charge through a conductor is the constant referred to in the previous equation and is known as the conductor's **resistance**. Ohm's Law is thus commonly stated algebraically as follows:

$$V = IR$$

We will see that this equation is the foundation of electrical circuit analysis.

The SI unit for resistance is the *ohm* (Ω). If a potential difference of 1 V across a conductor produces a current of 1 A, the resistance of the conductor is 1 Ω (1 Ω = 1 V/A).

The resistance of a material depends on several factors:

length	the longer the path, the greater the resistance
cross-sectional area	the smaller the cross-sectional area, the greater the resistance
material	different materials (*e.g.* aluminium, copper, iron) display different resistance characteristics
temperature	the higher the temperature, the greater the resistance

10.3.1.3.3 Electric Power

Electric power is a measure of the rate at which electrical energy is converted to other forms of energy, or the rate at which charge carriers do work. The electric power dissipated in a circuit is generally expressed algebraically as the product of the potential difference across, and the current in the circuit:

$$P = VI$$

Most light bulbs are labelled with their power ratings. The amount of heat and light given off by a bulb is related to the power rating, also known as the *wattage*, of the bulb.

Using Ohm's Law, we can express the power dissipated by a resistor in the following alternative forms:

$$P = IV = I(IR) = I^2R$$

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

The SI unit for electrical power is the *watt* (W), after the Scottish inventor James Watt (1736–1819).

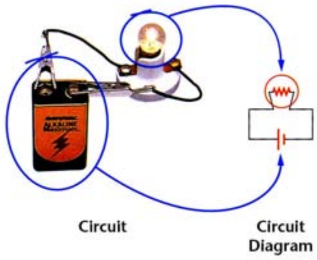
The unit of energy used by electric companies to calculate consumption, the *kilowatt-hour*, is defined in terms of power. One kilowatt-hour (kW·h) is the energy delivered in 1 h at the constant rate of 1 kW.

10.3.1.4 Circuits and Circuit Elements

10.3.1.4.1 Schematic Diagrams and Circuits

Schematic diagrams, or circuit diagrams, are a shorthand method of describing an electrical circuit, a path through which charges can be conducted.

An electrical circuit will generally comprise a source of charge or electrical energy, and some element that dissipates that energy. Any element of group of elements in a circuit that dissipates energy is called a **load**. A simple circuit, therefore, consists of a source of potential difference (electrical energy), such as a battery, and a load, such as a light bulb. Connecting wire and switches have negligible resistance and are generally not considered to be part of the load.



In many circuits, connecting wires are insulated (*i.e.* covered with a non-conductive material like plastic) and colour-coded. On a car battery for example, and also in building wiring, the wires connected to the positive terminal of a battery, or those in the *active circuit* of building wiring, have red insulation, while those from the negative terminal of a battery, or in the *neutral circuit* of building wiring, have black insulation. This distinction is not made in circuit diagrams.

Electrical components are usually labelled with their relevant ratings: light bulbs with a rated voltage (*i.e.* the maximum voltage drop that should be configured across the bulb) or wattage (more common for household light bulbs), resistors with their resistance, capacitors with their capacitance *etc.* Resistors, however, are unusual in that their rating is not labelled with text but rather with a number usually encoded in four coloured bands. The resistor colour code is presented in the following table:

Colour	1st Band	2nd Band	3rd Band	4th Band
Black	0	0	1	1
Brown	1	1	10	
Red	2	2	100	
Orange	3	3	1,000	
Yellow	4	4	10,000	
Green	5	5	100,000	
Blue	6	6	1,000,000	
Violet	7	7	10,000,000	
Grey	8	8	100,000,000	
White	9	9	1,000,000,000	
Gold			0.1	5%
Silver			0.01	10%
None				20%

To illustrate the use of this colour code, consider the typical resistor illustrated below. Resistance values are given as a number with two significant figures, encoded in the first two bands. A multiplier is encoded in the third band, and the fourth band indicates the tolerance (the accuracy of the labelled rating) of the device.

Reading from left to right:

1. The first band is brown, representing **1**;
2. The second band is black, representing **0**;
3. The third band is red, representing a multiplier of **100**, so that the resistance in question is 10 × 100 ohms, or **1000** Ω (also written **1**k Ω).
4. The fourth band is silver, indicating a tolerance of 10%, *i.e.* the resistor is guaranteed to have a resistance within 10% of the rated value of **1**k Ω.

10.3.1.4.2 Resistors in Series or in Parallel

In a circuit that consists of a single bulb and a battery, the potential difference across the bulb is equal to the terminal voltage. The total current in the circuit can be found using the equation $V = IR$.

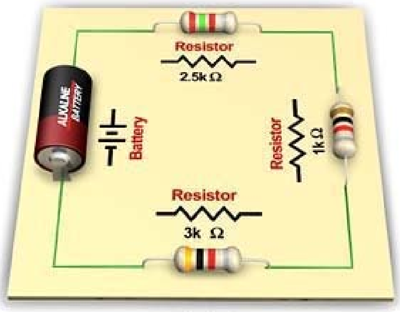
If there are several components in a circuit however, the voltage (potential difference), current and resistance at different points in the circuit will depend on the configuration of the individual components.

Resistors in Series

For resistors in series, the equivalent resistance is equal to the sum of the individual resistances:

$$R_{eq} = R_1 + R_2 + R_3 \ldots$$

and thus *the equivalent resistance of a series combination of resistors is always greater than any individual resistance.*



As an example, consider the adjacent circuit illustration. The equivalent resistance is given by:

$$\begin{aligned} R_{eq} &= (2.5 + 1 + 3)\text{k } \Omega \\ &= 6.5\text{k } \Omega \end{aligned}$$

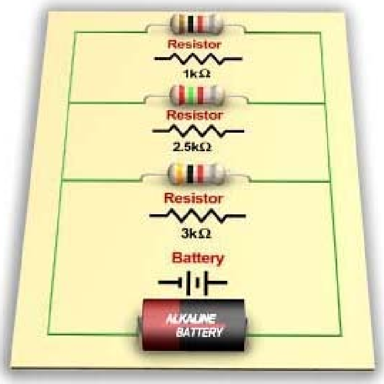
When resistors are connected in series, the current in each resistor is the same, and the potential difference across the series of resistors is equal to the sum of the potential differences across the individual resistors.

Resistors in Parallel

For resistors in parallel, the equivalent resistance can be calculated using a reciprocal relationship:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots$$

and thus *the equivalent resistance for a parallel arrangement of resistors must always be less than the smallest resistance in the group.*

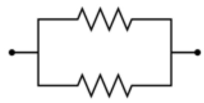


As an example, consider the adjacent circuit illustration. The equivalent resistance is given by:

$$\begin{aligned} R_{eq} &= \frac{1}{\left(\frac{1}{1} + \frac{1}{2.5} + \frac{1}{3}\right)} = \frac{1}{(1 + 0.4 + 0.33)} = \frac{1}{1.73} \\ &= 0.58\text{k } \Omega = 580\ \Omega \end{aligned}$$

The total current through a group of parallel resistors is equal to the sum of the currents in the individual resistors, and the potential difference across individual resistors connected in parallel is the same in each case.

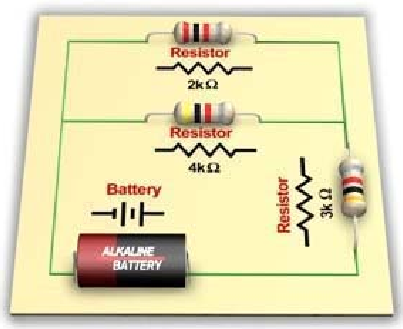
The rules for calculating current, voltage and resistance in circuits, often referred to as **Kirchhoff's circuit laws**, after the German physicist Gustav Kirchhoff (1824–1887) who first proposed them in 1845, are summarised in the following table.

Circuit Type	**Series**	**Parallel**
Schematic Diagram		
Current	$I = I_1 = I_2 = I_3 \cdots$ (same for each resistor)	$I = I_1 + I_2 + I_3 \cdots$ (sum of currents)
Potential Difference	$V = V_1 + V_2 + V_3 \cdots$ (sum of potential differences)	$V = V_1 = V_2 = V_3 \cdots$ (same for each resistor)
Equivalent Resistance	$R_{eq} = R_1 + R_2 + R_3 \cdots$ (sum of individual resistances)	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots$ (reciprocal of sum of resistances)

10.3.1.4.3 Resistors in Compound Circuits

Circuits often consist of combinations of series and parallel circuits. Such circuits are sometimes called *compound circuits*. To calculate the equivalent resistance in these situations, first isolate and simplify all branches of the circuit to their individual equivalent resistances. The following steps are helpful:

1. Calculate the equivalent resistances of resistors in parallel;
2. Calculate the equivalent resistances of resistors in series;
3. By repeating steps 1 and 2, as needed, the circuit can be simplified to an equivalent series circuit;
4. Simply add the equivalent resistances of the simplified equivalent series circuit to find the total resistance of the compound circuit.



As an example, consider the adjacent circuit illustration. The equivalent resistance can be calculated first calculating the equivalent resistance of the two resistors in parallel:

$$R_{eq} = \frac{1}{(\frac{1}{2}+\frac{1}{4})} = \frac{1}{(0.5+0.25)} = \frac{1}{0.75}$$
$$= 1.33\text{k } \Omega$$

At this point the circuit has been simplified to an equivalent series circuit consisting of an equivalent resistance of 1.33k Ω and a resistance of 3k Ω. The total resistance of the compound circuit can therefore be calculated as follows:

$$R_{eq} = (1.33+3)\text{k } \Omega$$
$$= 4.33\text{k } \Omega$$

References

Holt Physics, Serway, R.A. and Faughn, J.S. (Holt, Rinehart and Winston, 2000) [ISBN 0-03-056544-8] Ch. 17-22

Work directly from text, with exercises:

17 Electric Forces and Fields

17.1 Electric charge

17.2 Electric force

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18 Electrical Energy and Capacitance

18.1 Electrical potential energy

18.2 Potential difference

18.3 Capacitance

19 Current and Resistance

19.1 Electric current

19.2 Resistance

19.3 Electric power

20 Circuits and Circuit Elements

20.1 Schematic diagrams and circuits

20.2 Resistors in series or in parallel

20.3 Complex resistor combinations

The Molecular Expressions™ Web site, hosted by Florida State University, contains a lot of useful educational material on electricity and magnetism:

http://micro.magnet.fsu.edu/electromag/index.html